Merge Sort

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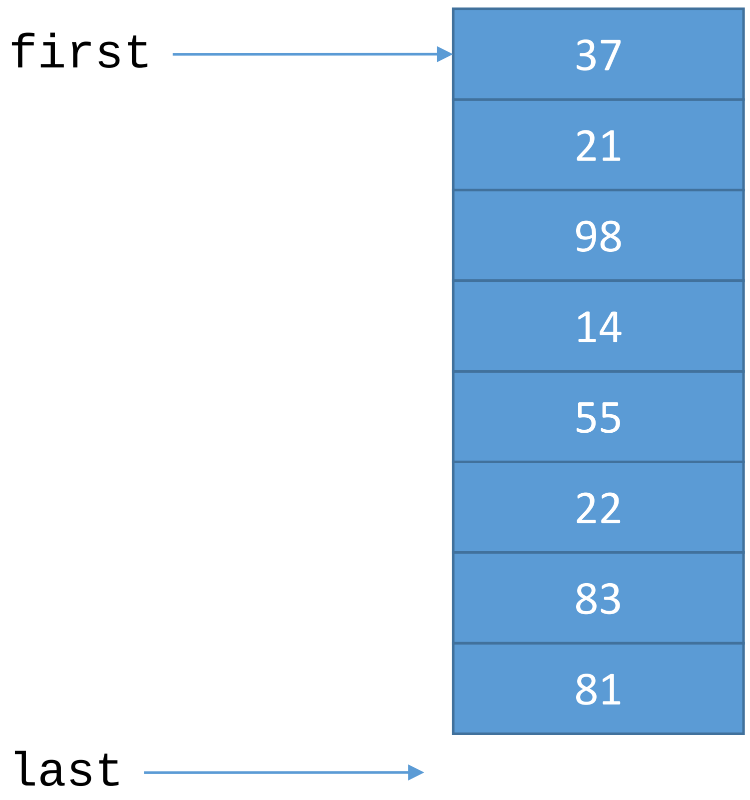
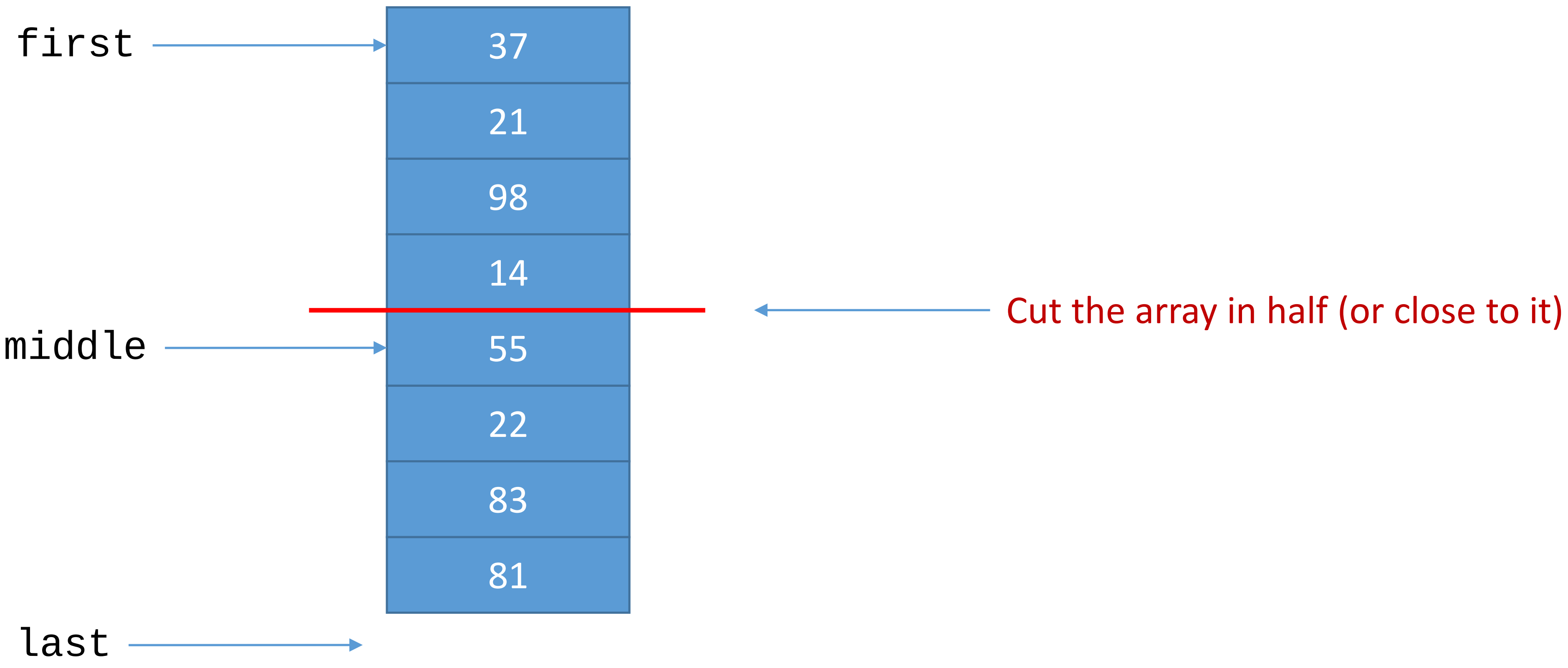
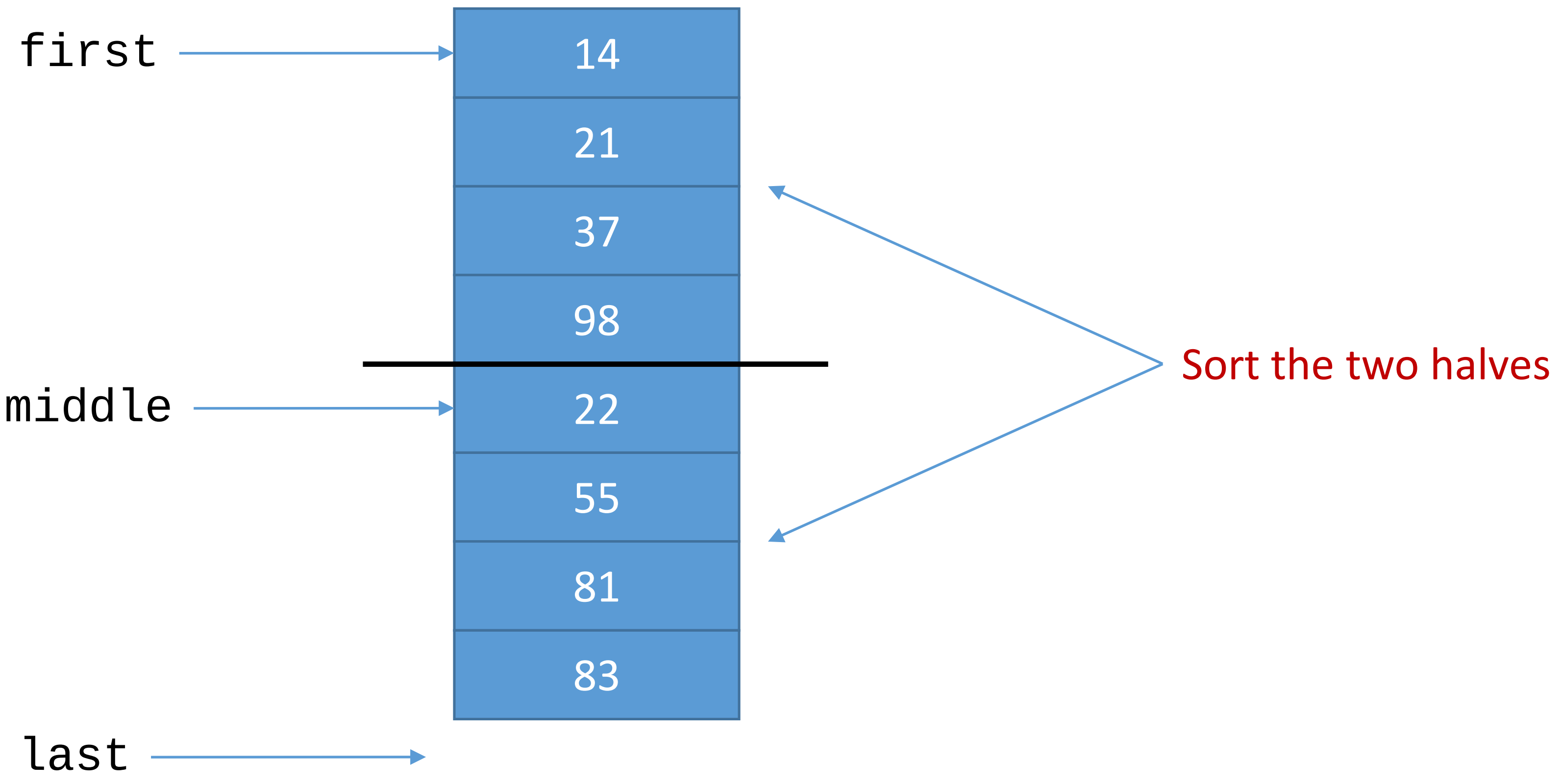
Starting Configuration



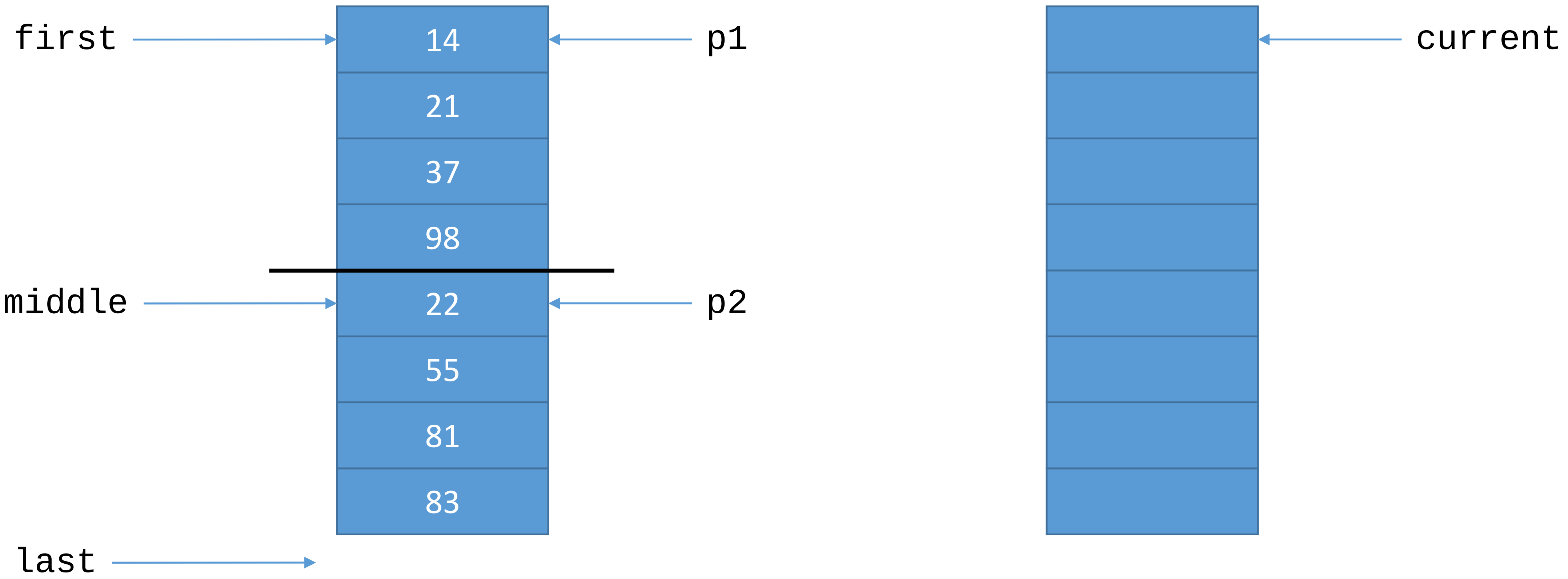
Split Into Subproblems



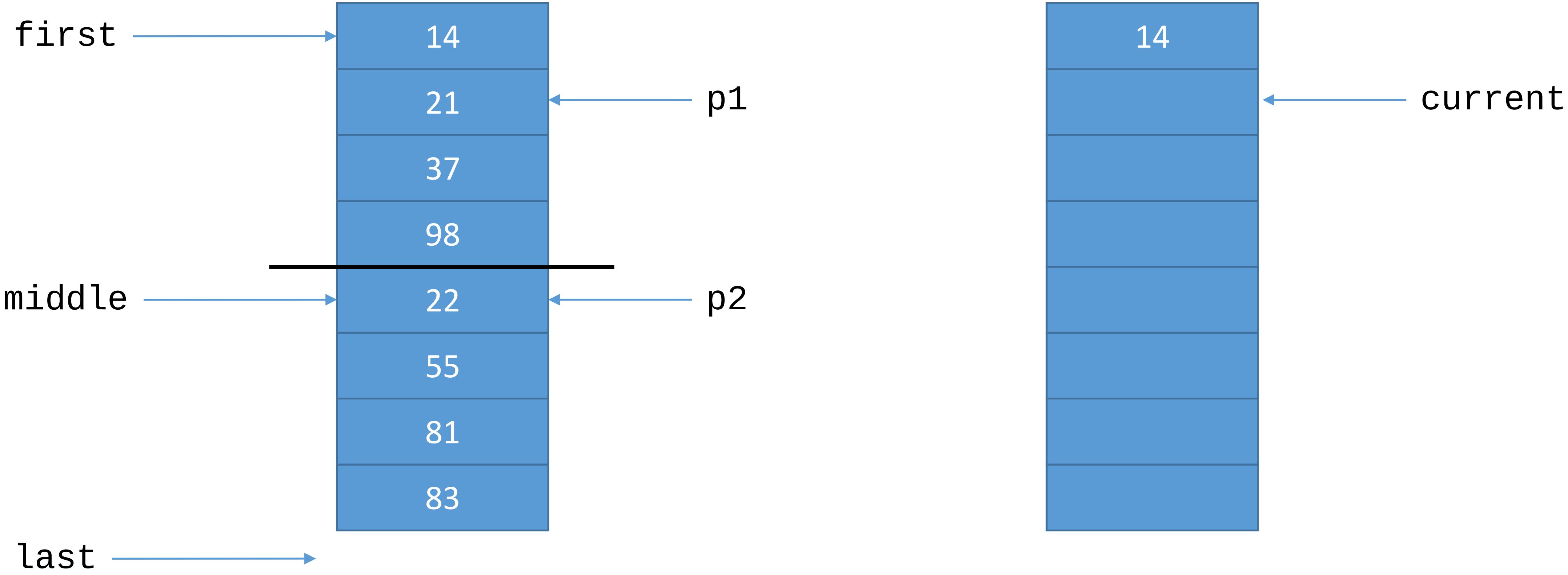
Recursively Solve Subproblems



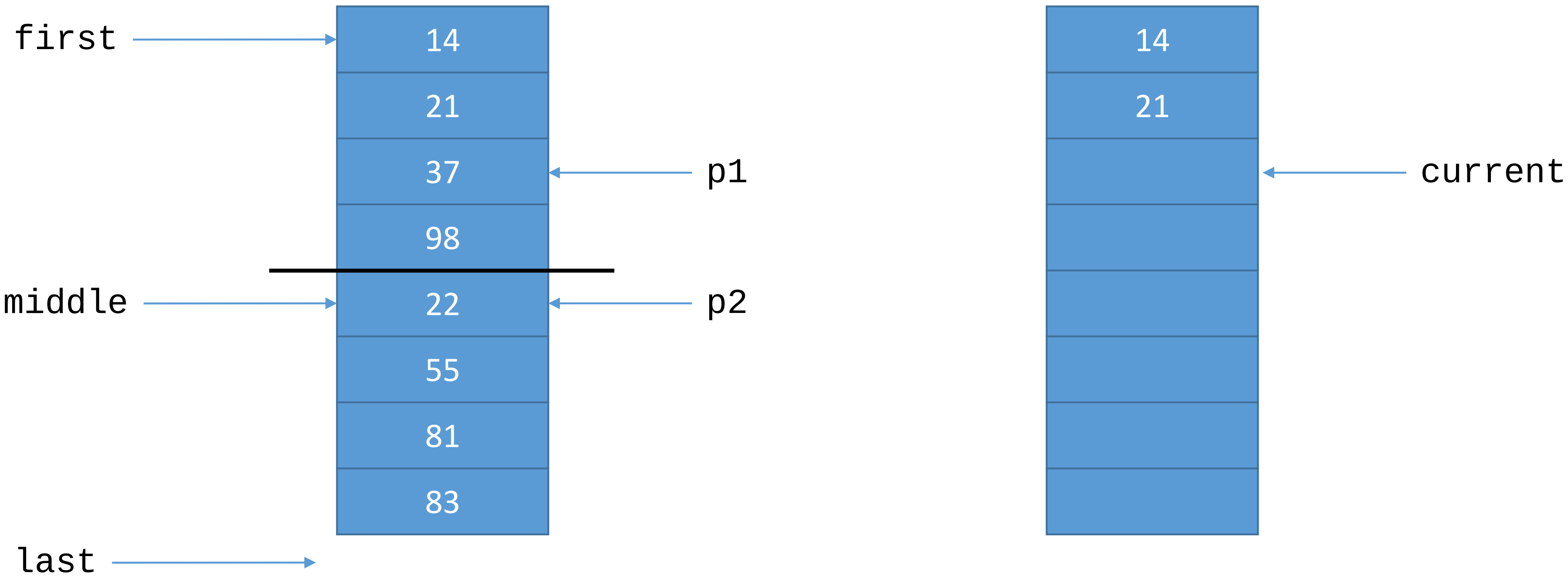
Merge the Subproblems



Merge the Subproblems



Merge the Subproblems



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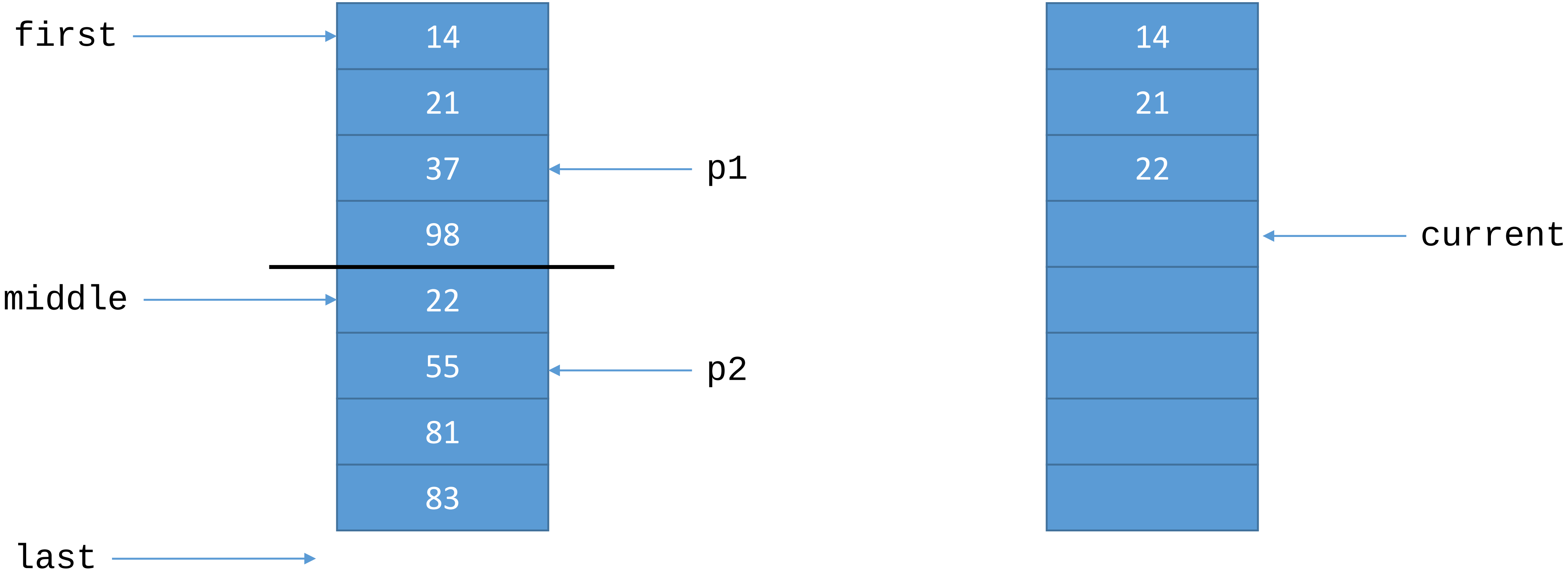

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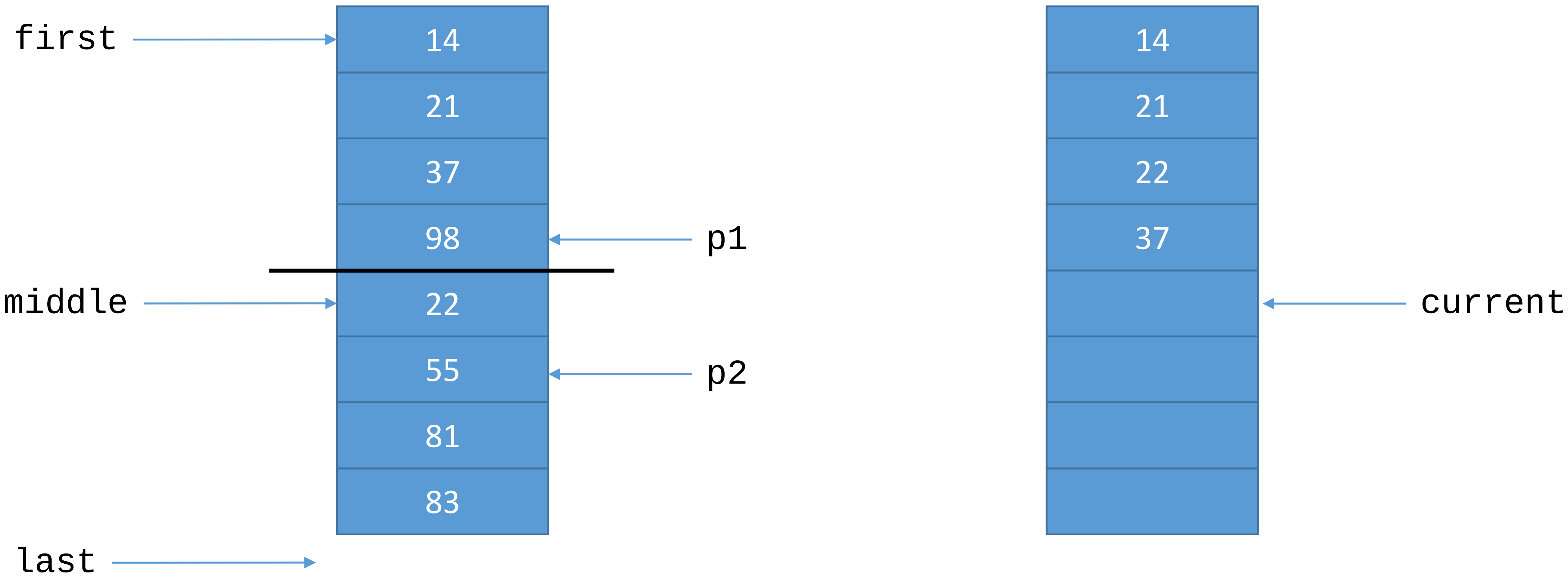

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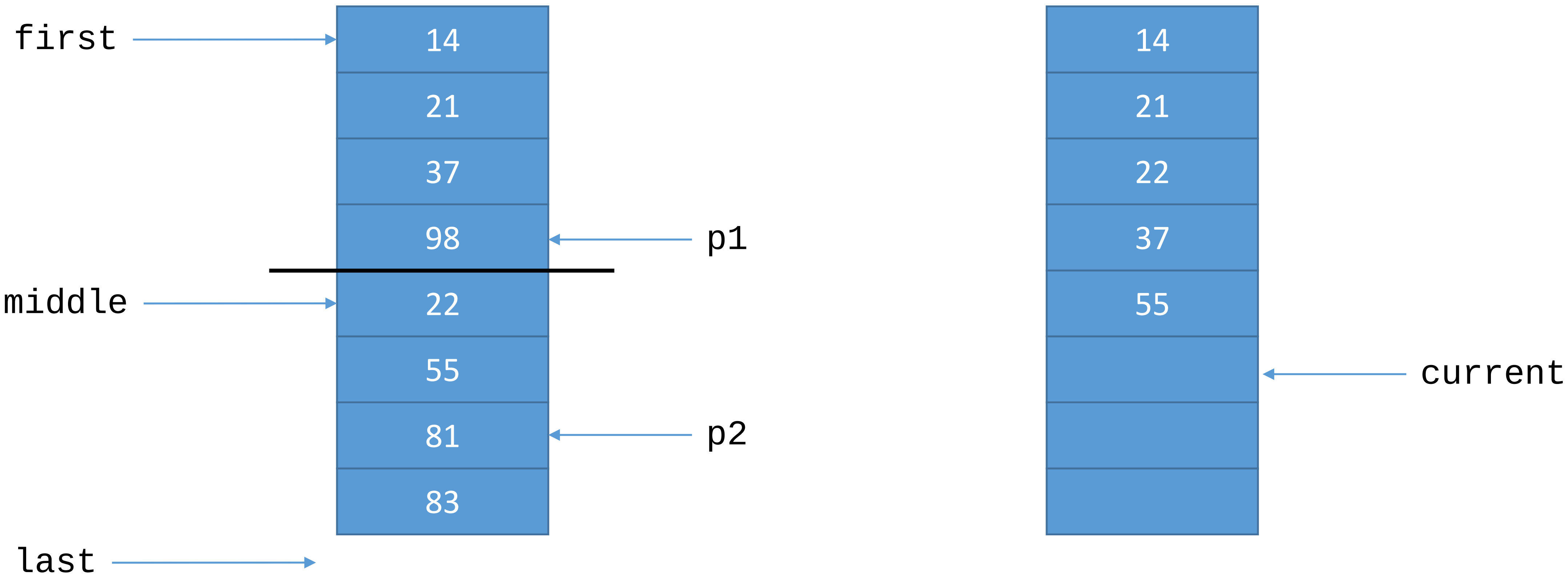

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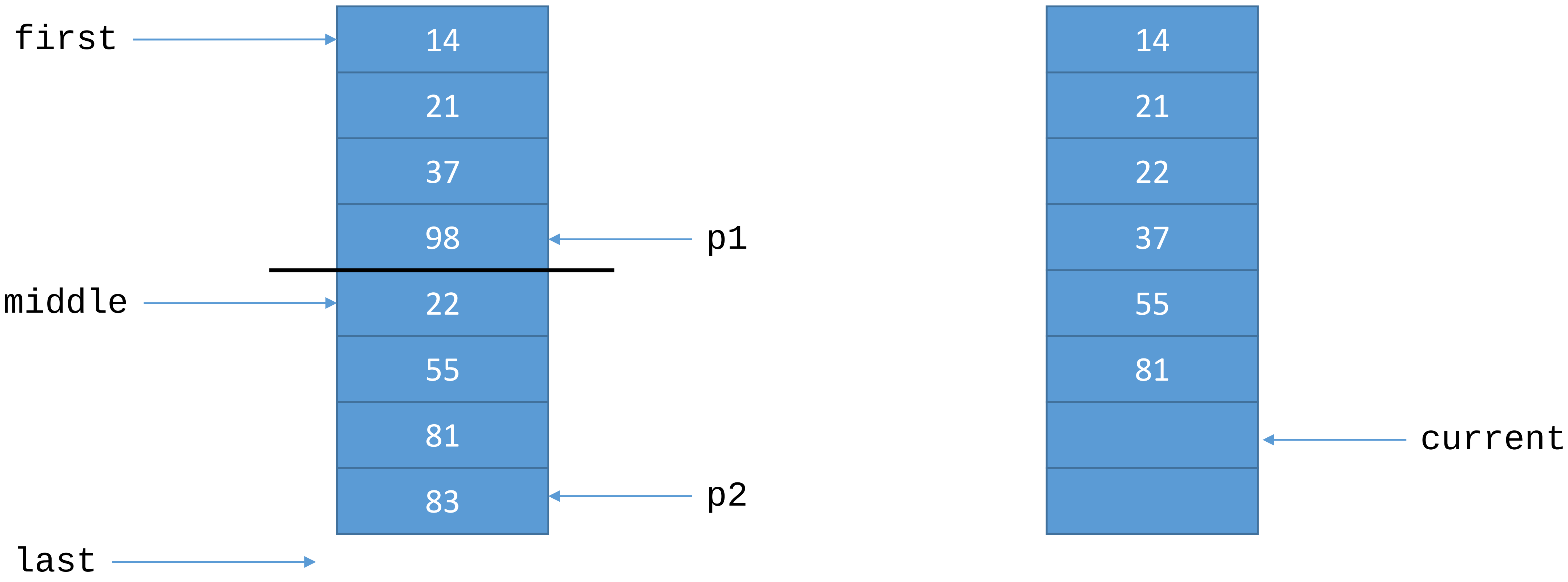

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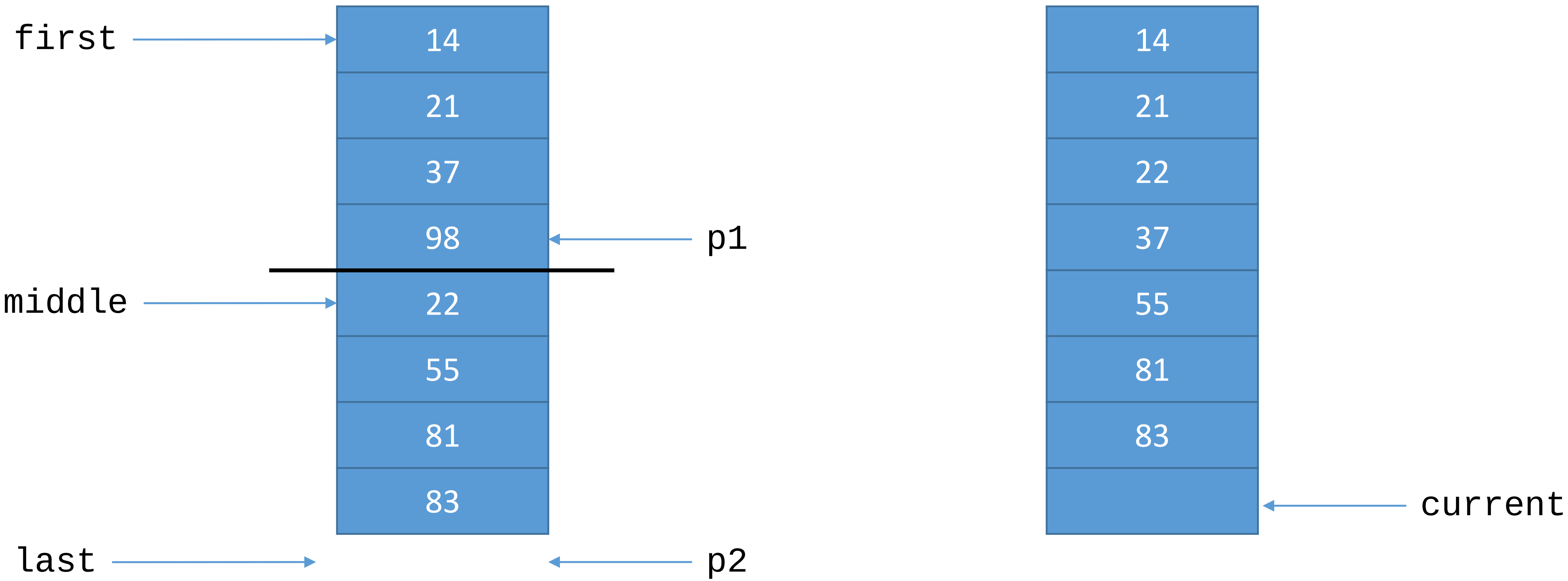

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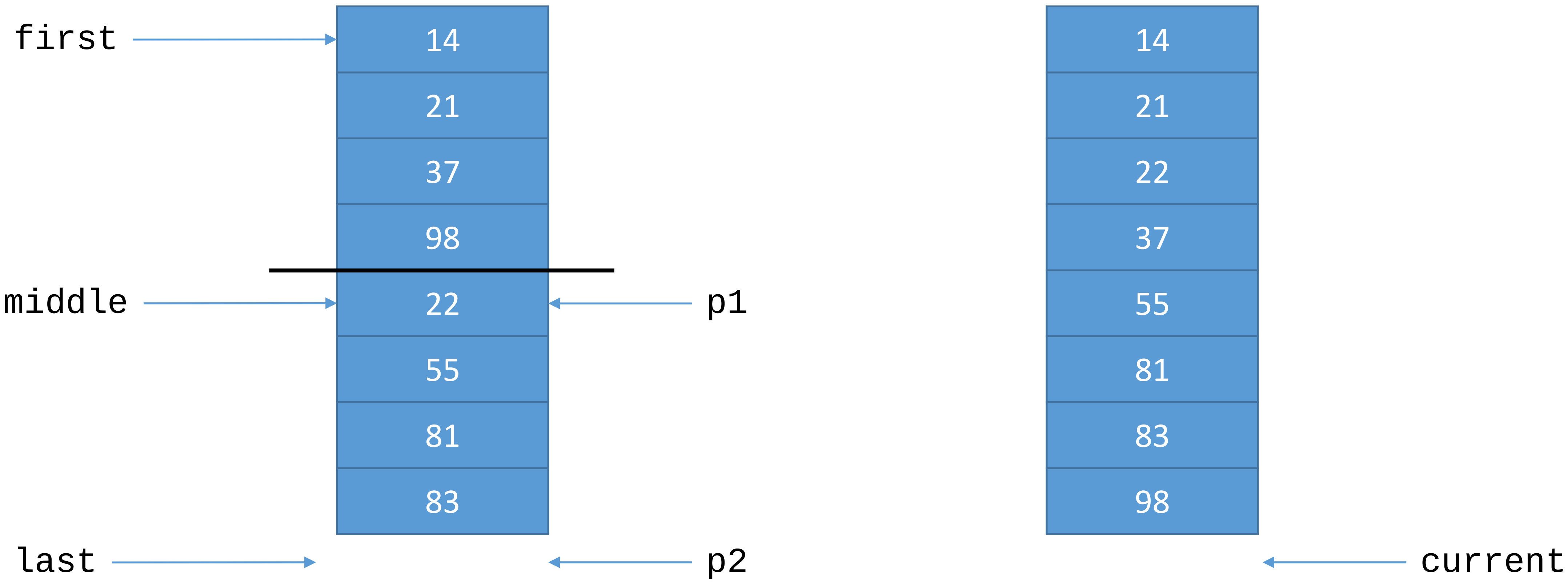
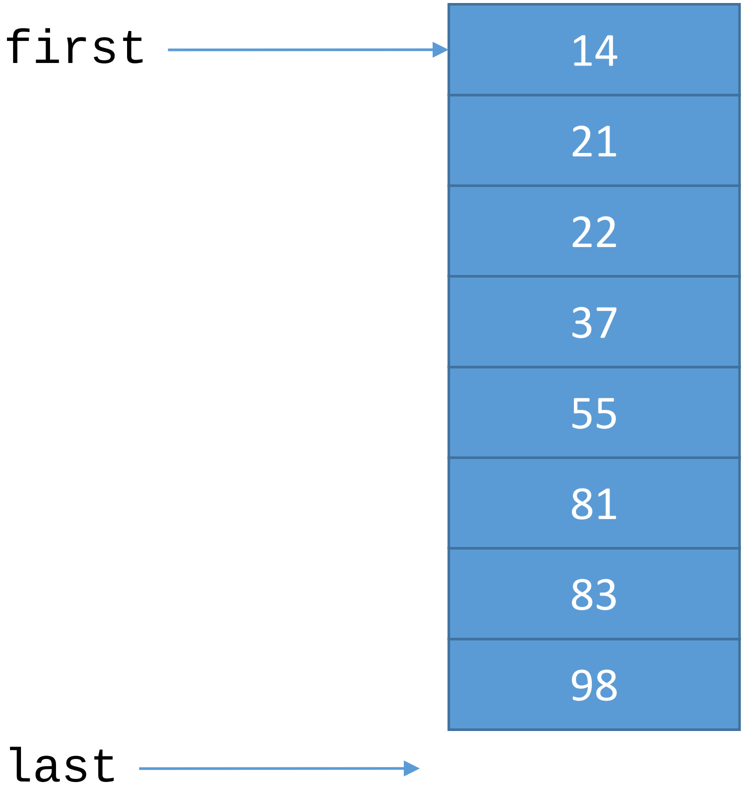


Copy Back



Pseudo-Code

```
IF <size of array is one or less> THEN
  <do nothing… array is already sorted>
ELSE
  <find midpoint of array>
  <sort first half>
  <sort second half>
  <merge the sorted halves together>
END IF
```

Requires allocating (and freeing!) a temporary array

Space and Time

- Merge Sort requires O(n) additional space beyond array.
  - Thus the method is expensive on space
  - Compare: Insertion Sort requires O(1) additional space!
- Time?
  - Not immediately obvious:
    - `T(n) = 2*T(n/2) + O(n)` ← Linear time to merge
    - A *recurrence* formula
  - Works out to O(n log(n))
  - Far superior to Insertion Sort's $O(n^2)$

Overhead of Recursion

- Using recursion down to subarray sizes of 1 is excessive
  - Huge overheads slow down the algorithm (though it remains O(n log(n))).
- Switch to another algorithm for small subarrays.

```
IF <size of array is less than threshold> THEN
  <use Insertion Sort on the array>
ELSE
  <find midpoint of array>
  <sort first half>
  <sort second half>
  <merge the sorted halves together>
END IF
```

Ideal threshold value found experimentally